◘ Based On Application Of Matrices & Determinants

LONG ANSWER TYPE QUESTIONS

Q01. (a) $x+y=4,\ 2x-3y=9$

By using matrix method $AX=B$, where $A=\begin{bmatrix}1 & 1\\ 2 & -3\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$, $B=\begin{bmatrix}4\\ 9\end{bmatrix}$

$\therefore |A|=\begin{vmatrix}1 & 1\\ 2 & -3\end{vmatrix}=-3-2=-5\neq 0 \qquad \Rightarrow A^{-1}$ exists.

Consider A_{ij} be the cofactor of corresponding element a_{ij} of the matrix A.

$A_{11}=-3,\quad A_{12}=-2,\quad A_{21}=-1,\quad A_{22}=1$

$\therefore \text{adj}\,A=\begin{bmatrix}-3 & -1\\ -2 & 1\end{bmatrix}$ As $A^{-1}=\frac{1}{|A|}\text{adj}(A)$ so, $A^{-1}=\frac{1}{-5}\begin{bmatrix}-3 & -1\\ -2 & 1\end{bmatrix}$

Since AX = B so, on Pre-multiplying by A^{-1}, we get : $A^{-1}AX=A^{-1}B \qquad \Rightarrow IX=A^{-1}B$

That is, $X=A^{-1}B \qquad \therefore \begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{+5}\begin{bmatrix}+3 & +1\\ +2 & +-1\end{bmatrix}\begin{bmatrix}4\\ 9\end{bmatrix}=\frac{1}{5}\begin{bmatrix}21\\ -1\end{bmatrix} \qquad \Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}21/5\\ -1/5\end{bmatrix}$

So, by using equality of matrices, we get : $x=21/5,\ y=-1/5$

(b) $-x+2y+3z=3,\ 2x+3y-2z=5,\ 3x+y+4z=11$

By using matrix method, AX = B, where $A=\begin{bmatrix}-1 & 2 & 3\\ 2 & 3 & -2\\ 3 & 1 & 4\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $B=\begin{bmatrix}3\\ 5\\ 11\end{bmatrix}$

Now $|A|=\begin{vmatrix}-1 & 2 & 3\\ 2 & 3 & -2\\ 3 & 1 & 4\end{vmatrix}=-1(14)-2(14)+3(-7)=-63\neq 0 \Rightarrow A^{-1}$ exists.

Now, let A_{ij} be the cofactors of corresponding elements a_{ij} of matrix A.

$A_{11}=14 \qquad A_{21}=-5 \qquad A_{31}=-13$

$A_{12}=-14 \qquad A_{22}=-13 \qquad A_{32}=4$

$A_{13}=-7 \qquad A_{23}=7 \qquad A_{33}=-7$

$\therefore \text{ajd}\,A=\begin{bmatrix}14 & -5 & -13\\ -14 & -13 & 4\\ -7 & 7 & -7\end{bmatrix}$ As $A^{-1}=\frac{1}{|A|}\text{adj } A$ so, $A^{-1}=\frac{1}{-63}\begin{bmatrix}14 & -5 & -13\\ -14 & -13 & 4\\ -7 & 7 & -7\end{bmatrix}$

Now AX = B $\qquad$ Pre-multiplying by A^{-1} both sides, $A^{-1}AX=A^{-1}B \qquad \Rightarrow X=A^{-1}B$

$\therefore X=\frac{1}{63}\begin{bmatrix}-14 & 5 & 13\\ 14 & 13 & -4\\ 7 & -7 & 7\end{bmatrix}\begin{bmatrix}3\\ 5\\ 11\end{bmatrix}=\frac{1}{63}\begin{bmatrix}-42+25+143\\ 42+65-44\\ 21-35+77\end{bmatrix} \qquad \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ 1\\ 1\end{bmatrix}.$

By equality of matrices, we get : $x=2,\ y=1,\ z=1$.

(c) $x+2y+z=7,\ x+3z=11,\ 2x-3y=1$

By using matrix method, AX = B where $A=\begin{bmatrix}1 & 2 & 1\\ 1 & 0 & 3\\ 2 & -3 & 0\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $B=\begin{bmatrix}7\\ 11\\ 1\end{bmatrix}$

Now $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{vmatrix} = 1(0+9) - 2(0-6) + 1(-3-0) = 9 + 12 - 3 = 18 \neq 0 \Rightarrow A^{-1}$ exists.

Consider A_{ij} be the cofactors of corresponding elements a_{ij} of matrix A.

$A_{11} = 9$ $\quad A_{21} = -3$ $\quad A_{31} = 6$

$A_{12} = 6$ $\quad A_{22} = -2$ $\quad A_{32} = -2$

$A_{13} = -3$ $\quad A_{23} = 7$ $\quad A_{33} = -2$

$\therefore \text{adj } A = \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$ $\qquad \Rightarrow A^{-1} = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$

Now $AX = B \Rightarrow X = A^{-1}B$ $\qquad \therefore X = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}\begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$ $\qquad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

By equality of matrices, we get : $x = 2, \ y = 1, \ z = 3$.

(d) Do yourself. Use $|A| = -2, \ A^{-1} = \frac{1}{-2}\begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$ and $x = 1, \ y = 2, \ z = 3$.

(e) $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2; \ x, y, z \neq 0$

By using matrix method, $AX = B$ where $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

Now $|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix} = 2(120-45) - 3(-80-30) + 10(36+36) = 150 + 330 + 720$

$\therefore |A| = 1200 \neq 0$ $\qquad \Rightarrow A^{-1}$ exists.

Consider A_{ij} be the cofactors of corresponding elements a_{ij} of matrix A.

$A_{11} = 75$ $\quad A_{21} = 150$ $\quad A_{31} = 75$

$A_{12} = 110$ $\quad A_{22} = -100$ $\quad A_{32} = 30$

$A_{13} = 72$ $\quad A_{23} = 0$ $\quad A_{33} = -24$

$\therefore \text{adj } A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$ $\qquad \therefore A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

Now, $AX = B$ $\qquad$ Pre-multiplying by A^{-1} we get : $A^{-1}AX = A^{-1}B$ $\qquad \therefore X = A^{-1}B$

$\Rightarrow X = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix}$ $\qquad \Rightarrow \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$

By using equality of matrices, $1/x = 1/2, 1/y = 1/3, 1/z = 1/5$ i.e., $x = 2, \ y = 3, \ z = 5$

(f) Do yourself. Use $|A| = -22$, $A^{-1} = \frac{1}{-22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$ and $x = 1,\ y = 2,\ x = 5$.

(g) $3x + \frac{4}{y} + 7xz = 14,\ 2x - \frac{1}{y} + 3xz = 4,\ x + \frac{2}{y} - 3xz = 0$.

By using matrix method, AX = B where $A = \begin{pmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{pmatrix}, X = \begin{pmatrix} x \\ 1/y \\ xz \end{pmatrix}, B = \begin{pmatrix} 14 \\ 4 \\ 0 \end{pmatrix}$

Now $|A| = \begin{vmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{vmatrix} = 3(3-6) - 4(-6-3) + 7(4+1) = -9 + 36 + 35 = 62 \neq 0 \ \therefore A^{-1}$ exists.

Consider A_{ij} be the cofactors of corresponding elements a_{ij} of matrix A.

$A_{11} = -3$ $A_{21} = 26$ $A_{31} = 19$

$A_{12} = 9$ $A_{22} = -16$ $A_{32} = 5$

$A_{13} = 5$ $A_{23} = -2$ $A_{33} = -11$

$\therefore \text{adj } A = \begin{pmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{pmatrix}$ $\therefore A^{-1} = \frac{1}{62}\begin{pmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{pmatrix}$

Now, AX = B   Pre-multiplying by A^{-1} we get : $A^{-1}AX = A^{-1}B$ $\therefore X = A^{-1}B$

$$\Rightarrow X = \frac{1}{62}\begin{pmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{pmatrix}\begin{pmatrix} 14 \\ 4 \\ 0 \end{pmatrix} = \frac{1}{62}\begin{pmatrix} -42+104+0 \\ 126-64+0 \\ 70-8+0 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ 1/y \\ xz \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

By using equality of matrices, $x = 1, 1/y = 1, xz = 1$ i.e., $x = 1,\ y = 1,\ z = 1$.

(h) $2x + y - z = 4...(i), 3x + y - 2z = 6....(ii), x - z = 2...(iii)$

By using matrix method, AX = B where $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}$

Now $|A| = \begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & -1 \end{vmatrix} = 2(-1-0) - 1(-3+2) - 1(0-1) = 0 \ \therefore A^{-1}$ does not exist.

$\therefore$ Given system of equations may be consistent (here, infinitely many solutions) or inconsistent (no trivial solutions) according as $(\text{adj } A)B = O$ or $(\text{adj } A)B \neq O$ respectively.

Consider A_{ij} be the cofactors of corresponding elements a_{ij} of matrix A.

$A_{11} = -1$ $A_{21} = 1$ $A_{31} = -1$

$A_{12} = 1$ $A_{22} = -1$ $A_{32} = 1$

$A_{13} = -1$ $A_{23} = 1$ $A_{33} = -1$

$\therefore \text{adj } A = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & -1 & -1 \end{bmatrix}$. Now $(\text{adj } A)B = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix} = \begin{bmatrix} -4+6-2 \\ 4-6+2 \\ -4+6-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = O$

Since $(\text{adj A})B = O$ so, the given system of equations is consistent with many solutions.

To obtain these solutions, put $z = k$ in (i) and (ii). We have $2x + y = 4 + k, 3x + y = 6 + 2k$

Consider $M = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, P = \begin{bmatrix} 4+k \\ 6+2k \end{bmatrix}$

$\because MX = P \quad \Rightarrow X = M^{-1}P \quad \Rightarrow X = \frac{1}{2-3}\begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4+k \\ 6+2k \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 4+k \\ 6+2k \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4-k+6+2k \\ 12+3k-12-4k \end{bmatrix} = \begin{bmatrix} 2+k \\ -k \end{bmatrix}$ By using equality of matrices, $x = 2+k,\ y = -k$.

Replacing these values of x, y, z in LHS of (iii), LHS : $x - z = (2+k) - k = 2 = \text{RHS}$

Since $x = 2+k,\ y = -k, z = k$ satisfy the given system so, $x = 2+k,\ y = -k, z = k$ where $k \in R$.

Note that this system of equations is **consistent with infinitely many solutions**. A few more such sums have been provided in Q13 (in this section).

Also note that this concept has been discussed in the **NCERT Textbook Part I** but, there's no question/example based on this concept in the Textbook.

Q02. Given $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{bmatrix} \therefore |A| = \begin{vmatrix} 1 & 2 & 5 \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix} = 1(1-4) - 2(-5) + 5(11) = 62 \neq 0 \Rightarrow A^{-1}$ exists

Consider A_{ij} be the cofactors of corresponding elements a_{ij} of matrix A.

$A_{11} = -3$ $\quad A_{21} = 22$ $\quad A_{31} = 7$

$A_{12} = 5$ $\quad A_{22} = -16$ $\quad A_{32} = 9$

$A_{13} = 11$ $\quad A_{23} = 2$ $\quad A_{33} = -5$

$\therefore \text{adj } A = \begin{bmatrix} -3 & 22 & 7 \\ 5 & -16 & 9 \\ 11 & 2 & -5 \end{bmatrix}$ So, $A^{-1} = \frac{1}{62}\begin{bmatrix} -3 & 22 & 7 \\ 5 & -16 & 9 \\ 11 & 2 & -5 \end{bmatrix} \ldots$ (i)

Consider the given system of equations : $x + 2y + 3z = 8,\ 2x - y + 2z = 8,\ 5x + y - z = 16$

These equations can be expressed as PX = B, where $P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 5 & 1 & -1 \end{bmatrix}, X = \begin{bmatrix} z \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 8 \\ 8 \\ 16 \end{bmatrix}$

So, $PX = B \quad \Rightarrow X = P^{-1}B \ldots$(ii) $\quad \left[\because P = A^T \Rightarrow P^{-1} = \left(A^T\right)^{-1} = \left(A^{-1}\right)^T\right.$

By (i) and (ii), $X = \frac{1}{62}\begin{bmatrix} -3 & 22 & 7 \\ 5 & -16 & 9 \\ 11 & 2 & -5 \end{bmatrix}^T \begin{bmatrix} 8 \\ 8 \\ 16 \end{bmatrix}$

$\Rightarrow X = \frac{1}{62}\begin{bmatrix} -3 & 5 & 11 \\ 22 & -16 & 2 \\ 7 & 9 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 8 \\ 16 \end{bmatrix}$ $\quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{62}\begin{bmatrix} -24+40+176 \\ 176-128+32 \\ 56+72-80 \end{bmatrix} = \begin{bmatrix} 96/31 \\ 40/31 \\ 24/31 \end{bmatrix}$

Using equality of matrices : $x = \frac{96}{31}, y = \frac{40}{31}, z = \frac{24}{31}$.

Q03. Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 2 & 3 \\ 2 & 2 & -1 \end{bmatrix}$ $\quad \therefore |A| = \begin{vmatrix} 2 & 0 & -1 \\ 1 & 2 & 3 \\ 2 & 2 & -1 \end{vmatrix} = 2(-8) - 1(-2) = -14 \neq 0 \Rightarrow A^{-1}$ exists.

Consider A_{ij} be the cofactors of corresponding elements a_{ij} of matrix A.

$A_{11} = -8 \quad A_{21} = -2 \quad A_{31} = 2$

$A_{12} = 7 \quad A_{22} = 0 \quad A_{32} = -7$

$A_{13} = -2 \quad A_{23} = -4 \quad A_{33} = 4$

So, $\text{adj}A = \begin{bmatrix} -8 & -2 & 2 \\ 7 & 0 & -7 \\ -2 & -4 & 4 \end{bmatrix} \qquad \therefore A^{-1} = \frac{1}{-14}\begin{bmatrix} -8 & -2 & 2 \\ 7 & 0 & -7 \\ -2 & -4 & 4 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 8 & 2 & -2 \\ -7 & 0 & 7 \\ 2 & 4 & -4 \end{bmatrix} \ldots\text{(i)}$

Now consider the given system of equations : $2x - z = 4,\ x + 2y + 3z = 0,\ 2x + 2y - z = 2$

These equations can be expressed as $PX = B$ where $P = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 2 & 3 \\ 2 & 2 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

So, $P^{-1}PX = P^{-1}B \quad \Rightarrow X = P^{-1}B \ldots\text{(ii)} \qquad [\because P = A \quad \Rightarrow P^{-1} = A^{-1}$

By (i) and (ii), $X = \frac{1}{14}\begin{bmatrix} 8 & 2 & -2 \\ -7 & 0 & 7 \\ 2 & 4 & -4 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \qquad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

Using equality of matrices : $x = 2,\ y = -1,\ z = 0$.

Q04. (a) Let the cost of one pen of variety 'A', 'B' and 'C' be x, y and, z (in ₹) respectively.
Therefore, we have $x + y + z = 21,\ 4x + 3y + 2z = 60,\ 6x + 2y + 3z = 70$.

Clearly $AX = B$ where $A = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $B = \begin{pmatrix} 21 \\ 60 \\ 70 \end{pmatrix}$.

Now $|A| = 1(5) - 1(0) + 1(-10) = -5 \neq 0 \quad \therefore A^{-1}$ exists.

Consider A_{ij} be the cofactor of corresponding element a_{ij} of matrix A.

$A_{11} = 5, \quad A_{21} = -1, \quad A_{31} = -1,$

$A_{12} = 0, \quad A_{22} = -3, \quad A_{32} = 2, \qquad \therefore A^{-1} = \frac{1}{|A|}.[\text{adj.}(A)] = \frac{1}{-5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$

$A_{13} = -10, \quad A_{23} = 4, \quad A_{33} = -1$

As $X = A^{-1}B \quad \Rightarrow \begin{pmatrix} x \\ y \\ x \end{pmatrix} = \frac{1}{-5}\begin{pmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{pmatrix}\begin{pmatrix} 21 \\ 60 \\ 70 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ 8 \end{pmatrix}$

By equality of matrices, $x = 5, y = 8, z = 8$.

Therefore, the cost of one pen of variety 'A', 'B' and 'C' is ₹ 5, ₹ 8 and, ₹ 8 respectively.

(b) Let the three numbers be x, y, z.

So $x + y + z = 6$, $y + 3z = 11$ and $x + z = 2y$ or $x - 2y + z = 0$

Now solve this system of equations yourself. Use $|A| = 9, A^{-1} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$,

Also the required numbers are : 1, 2, 3.

Q05. Do yourself. Use $|A| = -1,\ A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ and, $x - 1,\ y = 2,\ z = 3$.

Q06. We have $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$, $B=\begin{bmatrix}3&1&-1\\1&3&1\\-1&1&3\end{bmatrix}$ $\quad\therefore AB=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}\begin{bmatrix}3&1&-1\\1&3&1\\-1&1&3\end{bmatrix}$

$\Rightarrow AB=\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}$ $\qquad \Rightarrow AB=4I\ldots(i)\Rightarrow A\left(\frac{1}{4}B\right)=A^{-1}I\Rightarrow A^{-1}=\frac{1}{4}(B)$

Consider the given systems of equations : $2x-y+z=-1,\ -x+2y-z=4,\ x-y+2z=-3$

These equations can be expressed as $PX=D$ where $P=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, $D=\begin{bmatrix}-1\\4\\-3\end{bmatrix}$

So $X=P^{-1}D$ $\quad\left\{\begin{array}{l}\text{Also by (i), } AB=4I\Rightarrow A\left(\frac{1}{4}B\right)=I\ \therefore A^{-1}=\frac{1}{4}B\\ \because P=A\Rightarrow P^{-1}=A^{-1}=\frac{1}{4}\begin{bmatrix}3&1&-1\\1&3&1\\-1&1&3\end{bmatrix}\end{array}\right.$

$\therefore X=\frac{1}{4}\begin{bmatrix}3&1&-1\\1&3&1\\-1&1&3\end{bmatrix}\begin{bmatrix}-1\\4\\-3\end{bmatrix}$ $\quad\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\2\\-1\end{bmatrix}$ $\therefore$ by equality of matrices : $x=1,\ y=2,\ z=-1$

Q07. Let $A=\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}$, $B=\begin{bmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{bmatrix}$ $\quad\therefore AB=\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}\begin{bmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{bmatrix}$

$\Rightarrow AB=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $\qquad$ That is, $AB=I\ldots(i)$

Consider the given systems of equations : $x-y+2z=1,\ 2y-3z=1,\ 3x-2y+4z=2$

These equations can be expressed as : $PX=D$ where $P=\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, $D=\begin{bmatrix}1\\1\\2\end{bmatrix}$

Therefore, $X=P^{-1}D$ $\quad\left[\begin{array}{l}\text{By (i), } AB=I\Rightarrow A^{-1}=B\\ \because P=A\quad\therefore P^{-1}=A^{-1}=B\end{array}\right.$

So, $X=\begin{bmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{bmatrix}\begin{bmatrix}1\\1\\2\end{bmatrix}$ $\quad\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\5\\3\end{bmatrix}$ $\therefore$ by equality of matrices : $x=0,\ y=5,\ z=3$

Q08. Product of matrices $=8I$; $x=3, y=-2, z=-1$

Q09. Product of matrices $=-9I$; $x=3, y=2, z=-1$

Q10. $A^{-1}=-\frac{1}{63}\begin{bmatrix}14&-14&-7\\-5&-13&7\\-13&4&-7\end{bmatrix}$; $x=2, y=1, z=1$

Q11. $A^{-1} = -\frac{1}{17}\begin{bmatrix} -1 & -2 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$; $x = 1, y = 2, z = 3$

Q12. We have $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 2 & 0 \\ 3 & 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 2 & -2 \\ 2 & -4 & 2 \\ 0 & -6 & 4 \end{bmatrix}$ $\therefore AB = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 2 & 0 \\ 3 & 3 & -1 \end{bmatrix}\begin{bmatrix} -2 & 2 & -2 \\ 2 & -4 & 2 \\ 0 & -6 & 4 \end{bmatrix}$

$\Rightarrow AB = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} = -4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\Rightarrow AB = -4I$. Hence Verified.

Consider the given systems of equations : $2y - 2x - 2z = 0, 2x - 4y + 2z = 2, -6y + 4z = -8$

The equations can be expressed as : $PX = D$ where $P = \begin{bmatrix} -2 & 2 & -2 \\ 2 & -4 & 2 \\ 0 & -6 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $D = \begin{bmatrix} 0 \\ 2 \\ -8 \end{bmatrix}$

Therefore, $X = P^{-1}D$

By (i), $AB = -4I \Rightarrow \left(-\frac{1}{4}A\right)B = I$ $\therefore B^{-1} = -\frac{1}{4}A$

$\because P = B$ $\therefore P^{-1} = B^{-1} = -\frac{1}{4}A$

So, $X = -\frac{1}{4}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 2 & 0 \\ 3 & 3 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ -8 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5/2 \\ -1 \\ -7/2 \end{bmatrix}$

$\therefore$ by equality of matrices : $x = 5/2,\ y = -1,\ z = -7/2$.

Q13. (a) Given $2x + 3y = 5$...(i), $6x + 9y = 15$...(ii)

By using matrix method, AX = B where $A = \begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$

Now $|A| = \begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} = 2\times 9 - 6\times 3 = 0$ $\therefore A^{-1}$ does not exist.

$\therefore$ Given system of equations may be consistent (here, infinitely many solutions) or inconsistent (no solutions).

Consider A_{ij} be the cofactors of corresponding elements a_{ij} of matrix A.

$A_{11} = 9$, $A_{12} = -6$, $A_{21} = -3$, $A_{22} = 2$

$\therefore \text{adj } A = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$ Now $(\text{adj } A)B = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 15 \end{bmatrix} = \begin{bmatrix} 45 - 45 \\ -30 + 30 \end{bmatrix} = O$

Since $(\text{adj } A)B = O$ so, the given system of equations is consistent with many solutions.

To obtain these solutions, put $y = k$ in (i). We have $2x + 3k = 5$ $\Rightarrow x = \frac{5 - 3k}{2}$

Replacing these values of x and y in LHS of (ii), LHS : $6\left(\frac{5 - 3k}{2}\right) + 9k = 15 = \text{RHS}$

Since $x = \frac{5 - 3k}{2}$, $y = k$ satisfy the given system so, $x = 5 - 3k,\ y = 2k$ where $k \in R$.

(b) Proceed as in Q01 (h). Values of $x = \frac{7 - 16k}{11}, y = \frac{k + 3}{11}, z = k$

(c) $x = \frac{5}{3}, y = k - \frac{4}{3}, z = k$ (d) $x = k - 2, y = 8 - 2k, z = k$

(e) $x = \frac{1}{2} - k, y = k, z = 0$ (f) $x = \frac{17 - 5k}{7}, y = \frac{11k - 1}{7}, z = k$

(g) $x = \frac{k}{3}, y = \frac{2k}{3}, z = k$ (h) $x = 0, y = 0, z = 0$

Q14. Attempt yourself. Cost of wheat : ₹5, Cost of onion : ₹8, Cost of rice: ₹8.
City B spends more on onions.
Reasons for price rise : Less production, Black-marketing, Hoarding etc.
Measures to be taken : More production should be encouraged, black-marketers should be punished under the law etc.

Q15. Let the award money spent on the values of Tolerance, Kindness and Leadership be ₹x each, ₹y each and ₹z each respectively.
$\therefore$ 3x + 2y + z = 2200, 4x + y + 3z = 3100, x + y + z = 1200.

The given situation can be expressed as : $\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$

where $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$ $\Rightarrow AX = B$ $\Rightarrow X = A^{-1}B \ldots$(i)

Now, $|A| = 3(1 - 3) - 2(4 - 3) + 1\,(4 - 1) = -5 \neq 0$, so A^{-1} exists.
Consider A_{ij} as the cofactors of the element a_{ij} of matrix A.

$A_{11} = -2,\quad A_{12} = -1,\quad A_{13} = 3,$

$A_{21} = -1,\quad A_{22} = 2,\quad A_{23} = -1,$

$A_{31} = 5,\quad A_{32} = -5,\quad A_{33} = -5$

So, $adj.A = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$ $\therefore A^{-1} = \frac{1}{|A|} adj.A = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}$

By (i), $X = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 4400 + 3100 - 6000 \\ 2200 - 6200 + 6000 \\ -6600 + 3100 + 6000 \end{bmatrix} = \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$

By equality of matrices, we get : $x = 300, y = 400, z = 500$.
Hence the award money for the values of Tolerance, Kindness and Leadership is ₹300, ₹400 and ₹500 respectively.
Also, the value of **Obedience** can also be included for the awards.

Q16. Let the information about composition of the three families be given as follow :

No. of	Male	Female	Children
Family I	2	4	3
Family II	3	3	2
Family III	2	2	5

$= A$ (say)

$\because$ Saving by each family = (Earning of each member – Expenses by each member of family)

That is, $\begin{matrix} \text{Male} \\ \text{Female} \\ \text{Child} \end{matrix}\left\{\begin{bmatrix} 500 \\ 400 \\ 0 \end{bmatrix} - \begin{bmatrix} 300 \\ 250 \\ 40 \end{bmatrix}\right\} = \begin{bmatrix} 200 \\ 150 \\ -40 \end{bmatrix} = B$ (say)

$\therefore$ saving by each family per day $AB = \begin{bmatrix} 2 & 4 & 3 \\ 3 & 3 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 200 \\ 150 \\ -40 \end{bmatrix} = \begin{bmatrix} 400+600-120 \\ 600+450-80 \\ 400+300-200 \end{bmatrix} = \begin{bmatrix} 880 \\ 970 \\ 500 \end{bmatrix}$

So, ₹880, ₹970 and ₹500 is the saving by Family I, Family II and Family III respectively.
Necessity of saving : Saving is necessary for each family as in case of emergency our saving in good time helps us to survive in bad time.

Q17. **i.** It is evident that the three values honesty, punctuality and obedience have been represented by the symbols x, y and z respectively.

So, $\begin{pmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11000 \\ 10700 \\ 2700 \end{pmatrix}$

$\therefore\ 5x+4y+3z=11000,$
$4x+3y+5z=10700,$
$x+y+z=2700.$

ii. Let $A = \begin{pmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{pmatrix} \quad \Rightarrow |A| = \begin{vmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{vmatrix} = 5(3-5)-4(4-5)+3(4-3) = -3 \neq 0$

$\because A^{-1}$ exists, so equations have a unique solution.

iii. Value of **Honesty** should be most preferred as an honest student is most likely to be punctual and obedient.

Q18. Let the award money spent on the values of Discipline, Politeness and Punctuality be ₹x each, ₹y each and ₹z each respectively.
$\therefore$ 3x + 2y + z = 1000, 4x + y + 3z = 1500, x + y + z = 600.

The given situation can be expressed as : $\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix}$

where $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix} \quad \Rightarrow AX = B \quad \Rightarrow X = A^{-1}B \ldots(i)$

Now, $|A| = 3(1-3) - 2(4-3) + 1(4-1) = -5 \neq 0$, so A^{-1} exists.
Consider A_{ij} as the cofactors of the element a_{ij} of matrix A.

$A_{11} = -2, \quad A_{12} = -1, \quad A_{13} = 3,$
$A_{21} = -1, \quad A_{22} = 2, \quad A_{23} = -1,$
$A_{31} = 5, \quad A_{32} = -5, \quad A_{33} = -5$

So, $adj.A = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \qquad \therefore A^{-1} = \frac{1}{|A|} adj.A = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}$

By (i), $X = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2000+1500-3000 \\ 1000-3000+3000 \\ -3000+1500+3000 \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$

By equality of matrices, we get : $x = 100, y = 200, z = 300$.

Hence the award money for the values of Discipline, Politeness and Punctuality is ₹100, ₹200 and ₹300 respectively.
Also, the value of **Obedience** can also be included for the awards.

Q19. (i) Let x & y be the initial investments by Mr. Nakul Saini in bond A & bond B respectively.

Acc. to question, $\begin{bmatrix} x & y \\ x+\frac{20x}{100} & y+\frac{10y}{100} \end{bmatrix}\begin{bmatrix} 10\% \\ 15\% \end{bmatrix}=\begin{bmatrix} 4000 \\ 4500 \end{bmatrix} \Rightarrow \begin{bmatrix} x & y \\ \frac{12x}{10} & \frac{11y}{10} \end{bmatrix}\begin{bmatrix} 10\% \\ 15\% \end{bmatrix}=\begin{bmatrix} 4000 \\ 4500 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 3 \\ 8 & 11 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 80000 \\ 300000 \end{bmatrix}$ $\quad\therefore 2x+3y=80000,\ 8x+11y=300000$.

(ii) Let $A=\begin{bmatrix} 2 & 3 \\ 8 & 11 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \end{bmatrix}$, $B=\begin{bmatrix} 80000 \\ 300000 \end{bmatrix}$

$\therefore |A|=\begin{vmatrix} 2 & 3 \\ 8 & 11 \end{vmatrix}=22-24=-2\neq 0$, since $|A|\neq 0$ so A^{-1} exists. Therefore, it is possible to solve the system of linear equations by matrices.

Now, $A^{-1}=\frac{1}{-2}\begin{bmatrix} 11 & -3 \\ -8 & 2 \end{bmatrix}$ $\quad \because X=A^{-1}B=\frac{1}{2}\begin{bmatrix} -11 & 3 \\ 8 & -2 \end{bmatrix}\begin{bmatrix} 80000 \\ 300000 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 10000 \\ 20000 \end{bmatrix}$ $\quad$ Therefore, $x=$ ₹10000, $y=$ ₹20000.

Q20. Do yourself. (i) $x+y+z=70,\ 2x+3y+4z=210,\ 5y+4z=230$
(ii) $x=20,\ y=30,\ z=20$ $\quad$ (iii) Exercise keeps fit and healthy to a person.

Q21. **(a)** The cost per contact (in paise) is given in matrix A as $A=\begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix}\begin{matrix} \text{Telephone} \\ \text{House Call} \\ \text{Letters} \end{matrix}$

The number of contacts of each type made in two cities X and Y is given in the matrix B as

$$B=\begin{matrix} \text{Telephone} & \text{House Call} & \text{Letters} & \\ \end{matrix}$$
$$B=\begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix}\begin{matrix} \text{City X} \\ \text{City Y} \end{matrix}$$

The total amount spent by the party in the two cities $=BA$

That is, $$=\begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix}\begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix}=500\begin{bmatrix} 2 & 1 & 10 \\ 6 & 2 & 20 \end{bmatrix}\left(10\begin{bmatrix} 14 \\ 20 \\ 15 \end{bmatrix}\right)$$

$$\Rightarrow \quad =5000\begin{bmatrix} 2 & 1 & 10 \\ 6 & 2 & 20 \end{bmatrix}\begin{bmatrix} 14 \\ 20 \\ 15 \end{bmatrix}=5000\begin{bmatrix} 28+20+150 \\ 84+40+300 \end{bmatrix}=5000\begin{bmatrix} 198 \\ 424 \end{bmatrix}$$

$$\Rightarrow \quad =\begin{bmatrix} 990000 \\ 2120000 \end{bmatrix}\begin{matrix} \leftarrow \text{Amount spent in City X (in paise)} \\ \leftarrow \text{Amount spent in City Y (in paise)} \end{matrix}$$

Hence, the party spent 990000 paise (or, ₹9900) in the City X and, 2120000 paise (or, ₹21200) in the City Y.
One should consider party's **social activities** instead of promotional activities of the party before casting his/her vote.
(b) Do yourself. Cost per Contact: Telephone = ₹0.40, House calls = ₹1.00, Letters = ₹0.50. Telephone is better as it is cheap.

Q22. **(a)** Let the investment in first type of bond be ₹ x.
So, investment in second type bond shall be ₹ (35000 – x).

$\therefore \begin{pmatrix} x & 35000-x \end{pmatrix}\begin{pmatrix} 8\% \\ 10\% \end{pmatrix} = \begin{pmatrix} 3200 \end{pmatrix}$ $\Rightarrow \left(\frac{8x}{100} + \frac{350000-10x}{100}\right) = \begin{pmatrix} 3200 \end{pmatrix}$

By equality of matrices, $\frac{350000-2x}{100} = 3200$ $\Rightarrow 350000 - 2x = 320000$

$\therefore x = 15000$. Hence investment in 1st bond is ₹ 15000. And in 2nd bond, it's ₹ 20000.

(b) Do yourself. ₹15000 each type of bond. **Values reflected in the question:** (i) Charity (ii) Helping orphans or poor people (iii) Awareness about diseases.

Q23. Do yourself. x + y + z = 6, x + 2z = 7, 3x + y + z = 12 where x, y, z represent the number of students in categories I, II, III respectively. Also x = 3, y = 1, z = 2. Participating in co-curricular activities is very important. It is very essential for all round development.

Q24. (i) Combined Sale in September and October,

	Basmati	Permal	Naura	
A + B =	15000	30000	36000	Ramkrishna
	70000	40000	20000	Hari Prasad

(ii) Decrease in Sales in September to October,

	Basmati	Permal	Naura	
A − B =	5000	10000	24000	Ramkrishna
	30000	20000	0	Hari Prasad

(iii) Profit for each farmer = 2% of B $= 0.02 \times B$

	Basmati	Permal	Naura	
⇒ =	100	200	120	Ramkrishna
	400	200	200	Hari Prasad

Hence in October Ramkrishna receives ₹100, ₹200 & ₹120 as profit in the sale of each variety of rice respectively. Also Hari Prasad receives profit of ₹400, ₹200 & ₹200 in the sale of each variety of rice, respectively.

(iv) Hari Prasad gets more profit on sales of both varieties of rice for both the months.

(v) Hari Prasad is more resourceful as he makes more profit in same two months of time.

Q25. Let the amount deposited in the savings bank account with interest rates of 5%, 8% and $8\frac{1}{2}\%$ be x, y and z (in ₹) respectively.

So, $x + y + z = 7000, x = y, \frac{5x}{100} + \frac{8y}{100} + \frac{17z}{200} = 550$

i.e., $x + y + z = 7000, x - y = 0, 10x + 16y + 17z = 110000$.

On solving these by the help of matrices, we get : $x = 1125, y = 1125, z = 4750$.

So the amount deposited in different savings bank account with interest rates of 5%, 8% and 8 ½ % are ₹1125, ₹1125, and ₹4750 respectively.

Q26. Let the funds collected by schools A, B and C be x, y and z (in ₹) respectively.

So,
$$\begin{matrix} \text{Funds by school A} \rightarrow \\ \text{Funds by school B} \rightarrow \\ \text{Funds by school C} \rightarrow \end{matrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \overset{\text{Fans}\quad\text{Mats}\quad\text{Plates}}{\begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix}} \overset{\text{Cost}}{\begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 5 \times 25 \begin{bmatrix} 8 & 10 & 4 \\ 5 & 8 & 6 \\ 7 & 10 & 8 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} = 125\begin{bmatrix} 8+40+8 \\ 5+32+12 \\ 7+40+16 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 125\begin{bmatrix} 56 \\ 49 \\ 63 \end{bmatrix} = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$$

By equality of matrices, we get : $x = 7000,\ y = 6125,\ z = 7875$.

$\therefore$ the funds collected by school A is ₹7000, by school B is ₹6125 & by school C is ₹7875.

And, total funds collected = ₹(7000 + 6125 + 7875) = ₹21000.

Q27. The given information can be expressed as :

$$\begin{array}{l} \\ \text{Family A} \\ \text{Family B}\end{array}\begin{array}{c}\begin{array}{ccc}\text{Men} & \text{Women} & \text{Children}\end{array}\\ \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix}\end{array}\begin{array}{c}\begin{array}{cc}\text{Calories} & \text{Proteins}\end{array}\\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}\end{array}$$

$$\Rightarrow \begin{array}{l}\text{Family A} \\ \text{Family B}\end{array}\begin{bmatrix} 9600+11400+3600 & 180+330+66 \\ 4800+3800+7200 & 90+110+132 \end{bmatrix} = \begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix}$$

So 24600 calories and 576 grams of proteins are needed for Family A and 15800 calories and 332 grams of proteins are needed for Family B.

Balanced diet is of utmost importance for proper functionality and growth of our body. So people must be aware to take food which provides all the necessary nutrients.

Q28. The given information can be expressed as :

$$\begin{array}{l} \\ \text{Village X} \\ \text{Village Y} \\ \text{Village Z}\end{array}\begin{array}{c}\begin{array}{ccc}\text{Call} & \text{Letter} & \text{Announcement}\end{array}\\ \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}\end{array}\begin{array}{c}\text{Cost (in Rs.)}\\ \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}\end{array}$$

$$\Rightarrow \begin{array}{l}\text{Cost for village X} \\ \text{Cost for village Y} \\ \text{Cost for village Z}\end{array}\begin{bmatrix} 20000+6000+4000 \\ 15000+5000+3000 \\ 25000+8000+6000 \end{bmatrix} = \begin{bmatrix} 30000 \\ 23000 \\ 39000 \end{bmatrix}$$

Hence the cost incurred by organization for villages X, Y and Z respectively are 30000, 23000 and 39000 (in ₹).

Value of **cleanliness/ women welfare** is generated by the organization in society.

Q29. Let the rate of interest at which the half of the amount ₹30000 is deposited in the bank be x %.

$\therefore \begin{bmatrix} 2\% & x\% \end{bmatrix}\begin{bmatrix} 15000 \\ 15000 \end{bmatrix} = \begin{bmatrix} 1800 \end{bmatrix}$ $\quad \Rightarrow \begin{bmatrix} 15000 \times \frac{2}{100} + 15000 \times \frac{x}{100} \end{bmatrix} = \begin{bmatrix} 1800 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 300 + 150x \end{bmatrix} = \begin{bmatrix} 1800 \end{bmatrix}$ $\quad \Rightarrow 300 + 150x = 1800$ [By equality of matrices

$\Rightarrow 150x = 1800 - 300$ $\quad \Rightarrow x = \frac{1500}{150} = 10$

Hence the required rate of interest is 10%.

Yes, people should donate to the trusts which are involved in the service of society.

Q30. Do yourself. Ans. ₹7000, ₹11000, ₹6375.

Q31. Let the total amount given by the schools A, B and C be x, y and z (in ₹) respectively.

So,
$$\begin{array}{l}\text{Amount by school A} \rightarrow \\ \text{Amount by school B} \rightarrow \\ \text{Amount by school C} \rightarrow\end{array}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{array}{c}\begin{array}{ccc}\text{Honesty} & \text{Regualrity} & \text{Hardwork}\end{array}\\ \begin{bmatrix} 3 & 4 & 6 \\ 4 & 5 & 3 \\ 6 & 2 & 4 \end{bmatrix}\end{array}\begin{array}{c}\text{Cost}\\ \begin{bmatrix} 2500 \\ 3100 \\ 5100 \end{bmatrix}\end{array}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 50500 \\ 40800 \\ 41600 \end{bmatrix}$$

By equality of matrices, we get : $x = 50500,\ y = 40800,\ z = 41600$.

So the amount given by school A is ₹50500, by school B is ₹40800 and by school C is ₹41600.

Value : Respect for elders or any other relevant value.

Q32. Here $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}4 & -6\\ -2 & 4\end{bmatrix}$ $\Rightarrow AB=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}\begin{bmatrix}4 & -6\\ -2 & 4\end{bmatrix}=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}=2\,I$ ½

$\Rightarrow A\left(\frac{1}{2}B\right)=I \ \therefore A^{-1}=\frac{1}{2}B\ldots(i)$ 1

The given system of equations $2x+y=4,\ 3x+2y=1$ can be written as

$PX=C$ where, $P=\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$ and $C=\begin{bmatrix}4\\ 1\end{bmatrix}$

$\Rightarrow P^{-1}PX=P^{-1}C$ $\Rightarrow IX=P^{-1}C$ $\therefore X=P^{-1}C$ [Note that $P=A^T \therefore P^{-1}=(A^{-1})^T$ ½ + 1

Therefore, $X=\frac{1}{2}\begin{bmatrix}4 & -2\\ -6 & 4\end{bmatrix}\begin{bmatrix}4\\ 1\end{bmatrix}$ $\Rightarrow X=\frac{1}{2}\begin{bmatrix}14\\ -20\end{bmatrix}$ $\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}7\\ -10\end{bmatrix}$

By equality of matrices, we get : $x=7,\ y=-10$.

Q33. **(a)** Here $A=\begin{pmatrix}8 & 4 & 3\\ 2 & 1 & 1\\ 1 & 2 & 2\end{pmatrix}$.

Using $A=IA$, we have $\begin{pmatrix}8 & 4 & 3\\ 2 & 1 & 1\\ 1 & 2 & 2\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}A$

By $R_1\to R_1-7R_3$, $\begin{pmatrix}1 & -10 & -11\\ 2 & 1 & 1\\ 1 & 2 & 2\end{pmatrix}=\begin{pmatrix}1 & 0 & -7\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}A$

By $R_2\to R_2-2R_3$, $\begin{pmatrix}1 & -10 & -11\\ 0 & -3 & -3\\ 1 & 2 & 2\end{pmatrix}=\begin{pmatrix}1 & 0 & -7\\ 0 & 1 & -2\\ 0 & 0 & 1\end{pmatrix}A$

By $R_3\to R_3-R_1$, $\begin{pmatrix}1 & -10 & -11\\ 0 & -3 & -3\\ 0 & 12 & 13\end{pmatrix}=\begin{pmatrix}1 & 0 & -7\\ 0 & 1 & -2\\ -1 & 0 & 8\end{pmatrix}A$

By $R_3\to R_3+4R_2$, $\begin{pmatrix}1 & -10 & -11\\ 0 & -3 & -3\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}1 & 0 & -7\\ 0 & 1 & -2\\ -1 & 4 & 0\end{pmatrix}A$

By $R_2\to\left(-\frac{1}{3}\right)R_2$, $\begin{pmatrix}1 & -10 & -11\\ 0 & 1 & 1\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}1 & 0 & -7\\ 0 & -1/3 & 2/3\\ -1 & 4 & 0\end{pmatrix}A$

By $R_1\to R_1+10R_2$, $\begin{pmatrix}1 & 0 & -1\\ 0 & 1 & 1\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}1 & -10/3 & -1/3\\ 0 & -1/3 & 2/3\\ -1 & 4 & 0\end{pmatrix}A$

By $R_1\to R_1+R_3$, $\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}0 & 2/3 & -1/3\\ 0 & -1/3 & 2/3\\ -1 & 4 & 0\end{pmatrix}A$

By $R_2 \to R_2 - R_3$, $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$

As $I = A^{-1}A$, so $A^{-1} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix}$.

Now given equations are $8x + 4y + 3z = 19$, $2x + y + z = 5$, $x + 2y + 2z = 7$.

Let $P = \begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$, $Q = \begin{pmatrix} 19 \\ 5 \\ 7 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Since $PX = Q \quad \Rightarrow X = P^{-1}Q = A^{-1}Q \quad [\because A = P \quad \therefore P^{-1} = A^{-1}$

Therefore, $X = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix}\begin{pmatrix} 19 \\ 5 \\ 7 \end{pmatrix} \quad \Rightarrow X = \begin{pmatrix} 0 + \frac{10}{3} - \frac{7}{3} \\ 19 - \frac{65}{3} + \frac{14}{3} \\ -19 + 20 + 0 \end{pmatrix} \quad \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

By equality of matrices, we have : $x = 1, y = 2, z = 1$.

(b) Here $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$

Using elementary row operations, $A = IA$

$\Rightarrow \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \qquad$ By $R_1 \to R_1 - R_2$

$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \qquad$ By $R_2 \to R_2 - 2R_1$

$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \qquad$ By $R_2 \to R_2 - 4R_3$

$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ 0 & 0 & 1 \end{pmatrix} A \qquad$ By $R_3 \to R_3 + R_2$

$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{pmatrix} A$

Since we know that $I = A^{-1}A$ so, $A^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{pmatrix}$.

Now consider the equations $3x-3y+4z=21,\ \ 2x-3y+4z=20,\ \ -y+z=5$.

The matrix form of these equations is, $\begin{pmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}21\\20\\5\end{pmatrix}$

where $A=\begin{pmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{pmatrix}$, $X=\begin{pmatrix}x\\y\\z\end{pmatrix}$ and $B=\begin{pmatrix}21\\20\\5\end{pmatrix}$

So, $AX=B \Rightarrow X=A^{-1}B \quad \Rightarrow X=\begin{pmatrix}1&-1&0\\-2&3&-4\\-2&3&-3\end{pmatrix}\begin{pmatrix}21\\20\\5\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\-2\\3\end{pmatrix}$

Therefore, $x=1,\ y=-2,\ z=3$.

Q34. Let incomes of Aryan and Babban are 3x and 4x per month respectively. Also let their monthly expenditures are 5y and 7y per month respectively.
Since the monthly saving for both of them is ₹15,000 so, by using matrix :

$\begin{bmatrix}3x\\4x\end{bmatrix}-\begin{bmatrix}5y\\7y\end{bmatrix}=\begin{bmatrix}15000\\15000\end{bmatrix} \quad \Rightarrow \begin{bmatrix}3x-5y\\4x-7y\end{bmatrix}=\begin{bmatrix}15000\\15000\end{bmatrix}$

By equality of matrices : $3x-5y=15000...(i)$ and $4x-7y=15000...(ii)$

For solving (i) & (ii), let $A=\begin{pmatrix}3&-5\\4&-7\end{pmatrix}$, $B=\begin{pmatrix}15000\\15000\end{pmatrix}$ and $X=\begin{pmatrix}x\\y\end{pmatrix}$

Since $AX=B \ \Rightarrow X=A^{-1}B=\dfrac{1}{-21+20}\begin{pmatrix}-7&5\\-4&3\end{pmatrix}\begin{pmatrix}15000\\15000\end{pmatrix} \ \Rightarrow X=15000\begin{pmatrix}7&-5\\4&-3\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}$

$\Rightarrow \begin{pmatrix}x\\y\end{pmatrix}=15000\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}30000\\15000\end{pmatrix}$ By equality of matrices, we get : $x=30000,\ y=15000$.

Therefore, the monthly income for A and B are ₹90,000 and ₹1,20,000.
Value : **Saving in good time helps us survive in bad times.**

Q35. Let each poor child pay ₹ x per month and each rich child pay ₹ y per month.

Clearly, $20x+5y=9000...(i)$ and $5x+25y=26000...(ii)$

To solve (i) and (ii), let $A=\begin{pmatrix}20&5\\5&25\end{pmatrix}$, $B=\begin{pmatrix}9000\\26000\end{pmatrix}$, $X=\begin{pmatrix}x\\y\end{pmatrix}$ $\quad \because AX=B \ \Rightarrow X=A^{-1}B$

Now $A^{-1}=\dfrac{1}{500-25}\begin{pmatrix}25&-5\\-5&20\end{pmatrix}=\dfrac{1}{475}\begin{pmatrix}25&-5\\-5&20\end{pmatrix}$

$\therefore X=\dfrac{1}{475}\begin{pmatrix}25&-5\\-5&20\end{pmatrix}\begin{pmatrix}9000\\26000\end{pmatrix} \quad \Rightarrow \begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}200\\1000\end{pmatrix}$

Clearly $x=200,\ y=1000$.

Hence each poor child pays ₹ 200 per month and each rich child pays ₹ 1000 per month.
Value reflected : Helpfulness towards the needy and poor students.

Q36. Let the length and breadth of the of the land be x and y (in metres) respectively.
So, $(x-50)(y+50)=xy \Rightarrow 50x-50y=2500$ i.e., $x-y=50...(i)$

And, $(x-10)(y-20) = xy - 5300 \Rightarrow -20x - 10y = -5500$ i.e., $2x + y = 550$...(ii)

For solving (i) and (ii), let $A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 50 \\ 550 \end{pmatrix}$

$\because AX = B \quad \Rightarrow X = A^{-1}B \quad \Rightarrow X = \frac{1}{1+2}\begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 50 \\ 550 \end{pmatrix} \quad \left[\because A^{-1} = \frac{1}{|A|}\text{adj.}A\right.$

$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 50+550 \\ -100+550 \end{pmatrix} = \begin{pmatrix} 200 \\ 150 \end{pmatrix} \quad \therefore x = 200$ and $y = 150$

Hence dimensions of the land are : length = 200 m and breadth = 150 m.

Reason : Ishan knows the value of education, which may be the reason why he wishes to donate the plot for a school.

Q37. Let the charges of typing one English page be x and that of one Hindi page be ₹ y.

So, $10x + 3y = 145$...(i) and $3x + 10y = 180$...(ii)

To solve (i) and (ii), let $A = \begin{pmatrix} 10 & 3 \\ 3 & 10 \end{pmatrix}$, $B = \begin{pmatrix} 145 \\ 180 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ $\quad \because AX = B \quad \Rightarrow X = A^{-1}B$

Now $A^{-1} = \frac{1}{100-9}\begin{pmatrix} 10 & -3 \\ -3 & 10 \end{pmatrix} = \frac{1}{91}\begin{pmatrix} 10 & -3 \\ -3 & 10 \end{pmatrix} \quad \therefore X = \frac{1}{91}\begin{pmatrix} 10 & -3 \\ -3 & 10 \end{pmatrix}\begin{pmatrix} 145 \\ 180 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 15 \end{pmatrix}$

Clearly $x = 10,\ y = 15$.

Hence the charge of typing one English page is ₹ 10 and that of one Hindi page be ₹ 15.

So, typing cost of 5 Hindi pages would be normally ₹ 75.

But the poor boy was charged only ₹ 10. Therefore, the poor boy was charged ₹ 65 less.

Value reflected : Helpfulness towards the poor and needy people.

Q38. Let ₹ x be invested in first bond and ₹ y be invested in second bond.

The system of equation is : $\left.\begin{matrix} \frac{10x}{100} + \frac{12y}{100} = 2800 \\ \frac{12x}{100} + \frac{10y}{100} = 2700 \end{matrix}\right\} \Rightarrow \begin{matrix} 5x + 6y = 140000 \\ 6x + 5y = 135000 \end{matrix}$

Let $A = \begin{pmatrix} 5 & 6 \\ 6 & 5 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 140000 \\ 135000 \end{pmatrix}$. Since $AX = B \Rightarrow X = A^{-1}B$

$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-11}\begin{pmatrix} 5 & -6 \\ -6 & 5 \end{pmatrix}\begin{pmatrix} 140000 \\ 135000 \end{pmatrix} = \begin{pmatrix} 10000 \\ 15000 \end{pmatrix}$

By equality of matrices, $x = 10000,\ y = 15000$ $\quad \therefore$ total amount invested is ₹ 25000.

Value reflected here : **Care for elders**.