

# Practice Problem Series

## Based on Derivatives (Advanced Level)

❖ Differentiate the following functions with respect to  $x$  [Q01 – Q47] :

- Q01.  $3^{2\log_3 x}$       Q02.  $\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$ ,  $x \in (0, \pi) - \frac{\pi}{2}$       Q03.  $3^{x+3}$
- Q04.  $\log\left(\frac{1}{\sqrt{x}}\right)$       Q05.  $\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2$       Q06.  $x^a - a^x$
- Q07.  $\frac{2-3\sin x}{\cos x}$       Q08.  $e^{2x}(x^2 + \log 2x)$       Q09.  $\frac{x^3 \sin x}{\cos x}$
- Q10.  $x \sin x \log x$       Q11.  $x^n \log x$       Q12.  $\log_{x^2}(x)$
- Q13.  $\frac{x + \cos x}{x - \sin x}$       Q14.  $\frac{1 + \tan x^\circ}{1 - \tan x^\circ}$       Q15.  $\frac{\sec x + \tan x}{\sec x - \tan x}$
- Q16.  $\frac{\sin x - x \cos x}{x \sin x + \cos x}$       Q17.  $\frac{1}{px^2 + qx + r}$       Q18.  $\frac{x \tan x}{(\sec x + \tan x)}$
- Q19.  $\frac{x^2 - x + 1}{x^2 + x + 1}$       Q20.  $\frac{\sqrt{a} - \sqrt{x}}{\sqrt{a} + \sqrt{x}}$       Q21.  $\frac{10^x}{\tan x}$
- Q22.  $\frac{1 + \log x}{1 - \log x}$       Q23.  $\frac{1 + 5^x}{1 - 5^x}$       Q24.  $\sin(e^{x^2})$
- Q25.  $\log(\sec x + \tan x)$       Q26.  $e^{x \sin x} + \sin x^e$       Q27.  $e^{e^x} - \cos \sqrt{e^{2x}} + e^{3 \log x}$
- Q28.  $\log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$       Q29.  $\log \tan \frac{x}{2}$       Q30.  $\log \sqrt{\sin\left(\frac{x^2}{3} - 1\right)}$
- Q31.  $\log(x + \sqrt{x^2 + a^2})$       Q32.  $\log(x + \sqrt{x^2 - a^2})$       Q33.  $(x + \sqrt{x^2 + a^2})^n$
- Q34.  $\frac{\sqrt{a^2 + x^2} + \sqrt{a^2 - x^2}}{\sqrt{a^2 + x^2} - \sqrt{a^2 - x^2}}$       Q35.  $\sqrt{\frac{1-x}{1+x}}$       Q36.  $\log\left(\frac{\sin x}{1 + \cos x}\right)$
- Q37.  $\tan e^{\sin x}$       Q38.  $\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$       Q39.  $\sin^2 \log(3x - 2)$
- Q40.  $(\log \cos x)^2$       Q41.  $\cos \log \sin x$       Q42.  $\log x^x$
- Q43.  $x + 3^{\sqrt{\cot \log x}}$       Q44.  $\sin \sqrt{\tan x + \cot x}$       Q45.  $\cos\left(\frac{1-x^2}{1+x^2}\right)$
- Q46.  $\log_7 x$       Q47.  $\log_x x$
- Q48. If  $y = \sqrt{x^2 + a^2}$ , then show that  $y \frac{dy}{dx} - x = 0$ .
- Q49. If  $y = e^x + e^{-x}$ , then show that  $\frac{dy}{dx} = \sqrt{y^2 - 4}$ .

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Q50. If  $x\sqrt{1+y} + y\sqrt{1+x} = 0$ , then prove that  $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$ .

Q51. Find the rate at which the function  $f(x) = x^3 - 3x^2 + 5x - 7$  changes with respect to  $x$ .

Q52. If  $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ , show that  $\frac{dy}{dx} = y$ .

Q53. If  $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$  then, show that  $2xy \frac{dy}{dx} = \frac{x}{a} - \frac{a}{x}$ .

Q54. If  $y = \frac{x}{x+2}$ , prove that  $x \frac{dy}{dx} = y(1-y)$ .

Q55. Using induction, prove that  $\frac{d}{dx}(x^n) = nx^{n-1}$  for all  $n \in \mathbb{N}$ .

# Note that the derivative of logarithmic function (i.e.,  $\log x$ ) is not in the syllabus of class XI. However if you memorize  $\frac{d}{dx}(\log x) = \frac{1}{x}$  and consider the rules like chain rule, product rule etc. as you've studied, you could be able to solve the questions involving logarithmic functions. [By the way, it shall be coming up in class XII.]

## ❖ ANSWERS ❖

- Q01. Q01.  $2x$                       Q02.  $\sec^2 x$ , if  $0 < x < \frac{\pi}{2}$  and  $-\sec^2 x$ , if  $\frac{\pi}{2} < x < \pi$
- Q03.  $3^{x+3} \log_e 3$                       Q04.  $-\frac{1}{2x}$                       Q05.  $\cos x$
- Q06.  $ax^{a-1} - a^x \log_e a$                       Q07.  $\sec x(2 \tan x - 3 \sec x)$                       Q08.  $e^{2x} [2x^2 + 2 \log 2x + 2x + \frac{1}{x}]$
- Q09.  $x^3 \sec^2 x + 3x^2 \tan x$                       Q10.  $\sin x + x \cos x \log x + \sin x \log x$                       Q11.  $x^{n-1} [1 + n \log x]$
- Q12. 0                      Q13.  $\frac{1 - \cos x - \sin x + x \cos x - x \sin x}{(x - \sin x)^2}$                       Q14.  $\frac{\pi}{180} \sec^2 \left( \frac{\pi}{4} + x^\circ \right)$
- Q15.  $\frac{2 \cos x}{(1 - \sin x)^2}$                       Q16.  $\frac{x^2}{(x \sin x + \cos x)^2}$                       Q17.  $-\frac{2px + q}{(px^2 + qx + r)^2}$
- Q18.  $\frac{(\sec x + \tan x)(x \sec^2 x + \tan x) - x \sec x \tan x (\tan x + \sec x)}{(\sec x + \tan x)^2}$                       Q19.  $\frac{2(x^2 - 1)}{(x^2 + x + 1)^2}$
- Q20.  $-\sqrt{\frac{a}{x}} \cdot \frac{1}{[\sqrt{a} + \sqrt{x}]^2}$                       Q21.  $10^x (\cot x \cdot \log_e 10 - \operatorname{cosec}^2 x)$                       Q22.  $\frac{2}{x} \cdot \frac{1}{(1 - \log x)^2}$
- Q23.  $\frac{5^x \log_e 25}{(1 - 5^x)^2}$                       Q24.  $2xe^{x^2} \cos(e^{x^2})$                       Q25.  $\sec x$
- Q26.  $e^{x \sin x} (x \cos x + \sin x) + ex^{e-1} \cos x^e$                       Q27.  $e^{e^x + x} + \sqrt{e^{2x}} \sin \sqrt{e^{2x}} + 3x^2$
- Q28.  $\sec x$                       Q29.  $\operatorname{cosec} x$                       Q30.  $\frac{x}{3} \cot \left( \frac{x^2}{3} - 1 \right)$
- Q31.  $\frac{1}{\sqrt{x^2 + a^2}}$                       Q32.  $\frac{1}{\sqrt{x^2 - a^2}}$                       Q33.  $\frac{n(x + \sqrt{x^2 + a^2})^n}{\sqrt{x^2 + a^2}}$
- Q34.  $\frac{-4a^2 x}{\sqrt{a^4 - x^4}}$                       Q35.  $-\frac{1}{(1+x)\sqrt{1-x^2}}$                       Q36.  $\cot x + \frac{\sin x}{1 + \cos x}$
- Q37.  $e^{\sin x} \sec^2 e^{\sin x} \cos x$                       Q38.  $-\frac{8e^{4x}}{(e^{4x} - 1)^2}$                       Q39.  $\frac{3 \sin 2 \log(3x - 2)}{3x - 2}$
- Q40.  $-2 \tan x \log \cos x$                       Q41.  $-\cot x \sin \log \sin x$                       Q42.  $\log(e x)$
- Q43.  $1 - \frac{3^{\sqrt{\cot \log x}} (\log 3) \cdot [\operatorname{cosec}^2(\log x)]}{2x \sqrt{\cot \log x}}$                       Q44.  $\frac{\cos \sqrt{\tan x + \cot x}}{2\sqrt{\tan x + \cot x}} [\sec^2 x - \operatorname{cosec}^2 x]$
- Q45.  $\frac{4x}{(1+x^2)^2} \sin \left( \frac{1-x^2}{1+x^2} \right)$                       Q46.  $\frac{\log_7 e}{x}$                       Q47. 0                      Q51.  $3x^2 - 6x + 5.$